that have "effects" at the distance in question. Rather than describing particles and interactions in terms of unmeasurable parameters that describe ultra-high-energy behavior, we formulate observations in terms of the things that are actually relevant to the scales we might detect. The effective theory at any single distance scale doesn't go into the details of an underlying short-distance physical theory; it only asks about things you could hope to measure or see. If something is beyond the resolution of the scales at which you are working, you don't need its detailed structure. This practice is not scientific fraud, but a way of disregarding the clutter of superfluous information. It is an "effective" way to obtain accurate answers efficiently.

Everyone, including physicists, is happy to return to a three-dimensional universe when higher-dimensional details are beyond our resolution. Just as physicists will often treat a wire as if it is one-dimensional, we will also describe a higher-dimensional universe in lower-dimensional terms when the extra dimensions are minuscule and higher-dimensional details are too tiny to matter. Such a lower-dimensional description would summarize the observable effects of all possible higher-dimensional theories in which the extra dimensions are too tiny to see. For many purposes, such a lower-dimensional description is adequate, independent of the number, size, and shape of the additional dimensions.

The lower-dimensional quantities are not providing the fundamental description, but they are a convenient way of organizing observations and predictions. If you do know the short-distance details, or the microstructure, of a theory, you can use them to derive the quantities that appear in the low-energy description. Otherwise, those quantities are just unknowns to be experimentally determined.

The following chapter elaborates these ideas and considers the consequences of tiny rolled-up extra dimensions. The dimensions we'll consider first are minuscule, too tiny to make any difference at all. Later on, when we return to extra dimensions, we'll explore both the large and the infinite dimensions that recently radically revised this picture.

2

Restricted Passages: Rolled-up Extra Dimensions

No way out
None whatsoever.

Jefferson Starship

Athena awoke with a start. The previous day she had read Alice in Wonderland *and* Flatland *in order to seek some inspiration about dimensions. But that night she had the strangest dream, which, when fully conscious, she recognized as the result of having read the two books on the same day.**

Athena dreamed she had turned into Alice, slipped into a rabbit hole, and met the resident Rabbit, who had pushed her out into an unfamiliar world. Athena had thought it a rather rude way to convey a guest. Even so, she had eagerly looked forward to her upcoming adventure in Wonderland.

Athena was in for a disappointment, however. The resident Rabbit, who was fond of puns, had sent her instead to OneDLand, a strange, not so wonderful, one-dimensional world. Athena looked around—or, I should say, to her left and right—and discovered that all she could see were two points—one to her left and another to her right (but in a prettier color, she thought).

In OneDLand, all the one-dimensional people with their one-dimensional possessions were lined up along this single dimension like long, thin beads strung out along a thread. But even with her limited

*Or perhaps this story is a result of my having begun my education at the perhaps questionably named Lewis Carroll School, P.S. 179, in Queens.

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purview, Athena knew there must be more to OneDLand than met her eyes because of the outrageous din that met her ears. A Red Queen was well hidden behind a dot, but Athena couldn't miss her strident yells: "This is the most ridiculous chess game I have ever seen! I can't move any pieces, not even to castle!" Athena was relieved when she realized her one-dimensional existence shielded her from the wrath of the Red Queen.

But Athena's cozy universe did not last long. Slipping through a gap in OneDLand, she returned to the dreamworld's rabbit hole, which had an elevator that could take her to hypothetical, other-dimensional universes. Almost immediately, the Rabbit announced, "Next stop: TwoDLand—a two-dimensional world." Athena didn't think "TwoDLand" a very nice name, but she cautiously entered all the same.

Athena needn't have been so hesitant. Almost everything in TwoDLand looked the same as in OneDLand. She did notice one difference—a vial labeled "Drink me." Bored with one dimension, Athena promptly obeyed. She quickly shrank to a tiny size, and as she became smaller, a second dimension came into view. This second dimension was not very big—it was wrapped around in a fairly small circle. Her surroundings now resembled the surface of an extremely long tube. A Dodo was racing around this circular dimension, but he wanted to stop. So he kindly offered Athena, who looked rather hungry, some cake.

When Athena ate a morsel of the Dodo's dreamcake, she started to grow. After only a few bites (she was quite sure of this, as she was still rather hungry), the cake very nearly disappeared; all that remained was a very tiny crumb. At least Athena thought there was a crumb, but she could see it only when she squinted very hard. And the cake wasn't the only thing that had vanished from view: when Athena returned to her usual size, the entire second dimension had disappeared.

She thought to herself, "TwoDLand is very odd indeed. I'd best be getting home." Her return journey was not without further adventures, but those will be kept for another time.

Even if we don't know *why* three spatial dimensions are special, we can ask *how*. How is it possible that the universe could appear to have only three dimensions of space if the fundamental underlying spacetime contains more? If Athena is in a two-dimensional world, why does she sometimes see only one? If string theory is the correct description of nature, and there are nine dimensions of space (plus one of time), what has become of the missing six spatial dimensions? Why aren't they visible? Do they have any discernible impact on the world we see?

The last three questions are central to this book. However, the first order of business is to determine whether there is any way in which the evidence of extra dimensions can be hidden so that Athena's two-dimensional world would appear as one-dimensional, or a universe with extra dimensions would appear to have the three-spatial-dimensional structure we observe around us. If we're to accept the idea of a world with extra dimensions, whatever theory they come from, there must be a good explanation for why we have not yet detected even the slightest trace of their existence.

This chapter is about extremely small *compactified*, or rolled-up, dimensions. They don't extend for ever, like the three familiar dimensions; instead, they quickly loop back on themselves, like a tightly wound spool of thread. No two objects could be separated very far along a compactified dimension; any attempt at a long-distance excursion would instead turn into a journey that went round and round, like the Dodo's laps. Such compactified dimensions could be so small that we wouldn't ever notice their existence. Indeed, we'll see that if tiny rolled-up dimensions exist, they will be quite a challenge to detect.

Rolled-up Dimensions in Physics

String theory, the most promising candidate for a theory combining quantum mechanics and gravity, gives a concrete reason to think about extra dimensions: the only coherent versions of string theory that we know of are laden with these surprising appendages. However, although the arrival of string theory in the physics world improved

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the respectability of extra dimensions, the idea of extra dimensions originated much earlier.

Back in the early twentieth century, Einstein's theory of relativity opened the door to the possibility of extra dimensions of space. His theory of relativity describes gravity, but it doesn't tell us why we experience the particular gravity we do. Einstein's theory does not favor any particular number of spatial dimensions. It works equally well for three or four or ten. Why, then, do there seem to be only three?

In 1919, close on the heels of Einstein's theory of general relativity (completed in 1915), the Polish mathematician Theodor Kaluza recognized this possibility in Einstein's theory and boldly proposed a fourth spatial dimension, a new unseen dimension of space.* He suggested that the extra dimension somehow might be distinguished from the three familiar infinite ones, though he didn't specify how. Kaluza's goal with this extra dimension was to unify the forces of gravity and electromagnetism. Although the details of that failed unification attempt are irrelevant here, the extra dimension that he had so brazenly introduced is very relevant indeed.

Kaluza wrote his paper in 1919. Einstein, who was the referee evaluating it for publication in a scientific journal, wavered about the merits of the idea. Einstein delayed the publication of Kaluza's paper for two years, but eventually acknowledged its originality. Yet Einstein still wanted to know what this dimension was. Where was it and why was it different? How far did it extend?

These are the obvious questions to ask. They might be some of the very same questions that are bothering you. No one responded to Einstein until 1926, when the Swedish mathematician Oskar Klein addressed his questions. Klein proposed that the extra dimension would be curled up in the form of a circle, and that it would be extremely small, just 10^{-33} cm,† one thousandth of a millionth of a

*We will specify spatial dimensions in this and the following chapter. After introducing relativity, we will switch to spacetime, and consider time as an additional dimension.

†I will sometimes use scientific notation for very large or very small numbers. When a power of ten has a negative exponent, as in 10^{-33}, it indicates a decimal number; for example, 10^{-33} is the number 0.000,000,000,000,000,000,000,000,000,000,001.

trillionth of a trillionth of a centimeter. This tiny rolled-up dimension would be everywhere: each point in space would have its own minuscule circle, 10^{-33} cm in size.

This small quantity represents the Planck length, a quantity that will be relevant later when we discuss gravity in more detail. Klein picked the Planck length because it is the only length that could naturally appear in a quantum theory of gravity, and gravity is connected to the shape of space. For now, all you need to know about the Planck length is that it is extraordinarily, unfathomably small—far smaller than anything we would ever have a chance of detecting. It is about twenty-four orders of magnitude† smaller than an atom and nineteen orders of magnitude smaller than a proton. It's easy to overlook anything as tiny as that.

There are many examples in daily life of objects whose extent in one of the three familiar dimensions is too small to be noticed. The paint on a wall, or a clothesline viewed from far away, are examples of things that seem to extend in fewer than three dimensions. We overlook the paint's depth and the clothesline's thickness. To a casual observer, the paint looks as if it has only two dimensions, and the clothesline appears to have only one, even though we know that actually both have three. The only way to see the three-dimensional structure of such things is to look up close, or with sufficiently fine resolution. If we stretched a hose across a football field and viewed it from a helicopter above, as is illustrated in Figure 15, the hose would look one-dimensional. But up close, you can resolve the two dimensions of the hose's surface and the three-dimensional volume it encloses.

For Klein, though, the thing that was undiscernibly small was not the thickness of an object, but a dimension itself. So what does it mean for a dimension to be small? What would a universe with a

This is an extremely tiny number and would be too cumbersome to write in full each time it occurs. A number with a positive exponent, such as 10^{33}, has 33 zeroes after a 1, 1,000,000,000,000,000,000,000,000,000,000,000, which is an enormous number that would also be difficult to write in full each time. I will often give a number in both scientific notation and in words the first time I use it.

†An order of magnitude is a factor of ten. Twenty-four orders of magnitude is 1,000,000,000,000,000,000,000,000, or one trillion trillion.

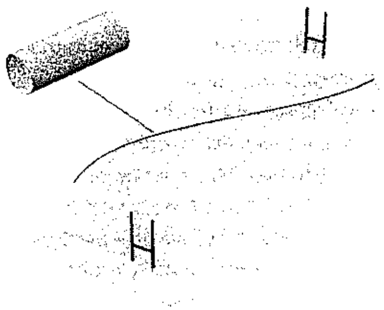

Figure 15. *When you view a hose spread over a football field from above, it looks like it has one dimension. But when you view it up close, you see that the surface has two dimensions and the volume it encloses has three.*

curled-up dimension look like to someone living inside it? Once again, the answer to these questions depends entirely on the size of the curled-up dimension. Let's consider an example to see what the world would look like to conscious beings that are small or big compared with the size of a rolled-up extra dimension. Because drawing four or more spatial dimensions is impossible, the first picture I'll present of a universe with a small, compactified dimension will have only two dimensions, with one of them rolled up tightly to a very small size (see Figure 16).

Figure 16. *When one dimension is curled up, a two-dimensional universe looks one-dimensional.*

Imagine again a garden hose, which can be thought of as a long sheet of rubber rolled up into a tube with a small circular cross-section. This time, we'll think of the hose as the entire universe (not an object

inside the universe).* If the universe were shaped like this garden hose, we would have one very long dimension and one very small, rolled-up dimension—exactly what we want.

For a little creature—a flat bug, say—that lived in the garden-hose universe, the universe would look two-dimensional. (In this scenario, our bug has to stick to the surface of the hose—the two-dimensional universe doesn't include the interior, which is three-dimensional.) The bug could crawl in two directions: along the length of the hose or around it. Like the Dodo, who could run laps in its two-dimensional universe, a bug that started somewhere along the hose could crawl around and eventually return to where it started. Because the second dimension is small, the bug wouldn't travel very far before it returned.

If a population of bugs living on the hose experienced forces, such as the electric force or gravity, those forces would be able to attract or repel bugs in any direction on the surface of the hose. Bugs could be separated from one another either along the length of the hose or around the hose's circumference, and would experience any force that was present on the hose. Once there is sufficient resolution to distinguish distances as small as the diameter of the hose, forces and objects exhibit both of the dimensions they actually have.

However, if our bug could observe its surroundings, it would notice that the two dimensions were very different. The one along the length of the hose would be very big. It could even be infinitely long. The other dimension, on the other hand, would be very small. Two bugs could never get very far from each other in the direction around the hose. And a bug that tried to take a long trip in that direction would quickly end up back where it started. A thoughtful bug that liked to stretch its legs would know that its universe was two-dimensional, and that one dimension extended a long way while the other was very small and rolled up into a circle.

But the bug's perspective is nothing like the one that creatures like

*The garden hose has always been a popular analogy to illustrate rolled-up dimensions. I learned it at math camp and it has most recently been described in Brian Greene's *Elegant Universe* (Norton, 1999; Vintage, 2000). I'll use this same analogy since it's so good and because I want to expand on it in the following section (and in later chapters), in which I'll also include sprinklers to explain extra-dimensional gravity.

us would have in Klein's universe, in which the extra dimension is rolled up to an extremely small size, 10^{-33} cm. Unlike the bug, we are not small enough to detect—never mind travel in—a dimension of such a tiny size.

So to complete our analogy, suppose that something much bigger than a bug, capable only of much coarser resolution and therefore unable to detect small objects or structure, lived in the garden-hose universe. Since the lens through which this bigger being views the world blurs any details that are as small as the hose's diameter, from the vantage point of this bigger being the extra dimensions would be invisible. It would see only a single dimension. Someone would see that the garden-hose universe had more than a single dimension only if he had sufficiently sharp vision to register something as small as the width of the hose. If his vision is too fuzzy to register that width, all he'll ever notice is a line.

Moreover, physical effects wouldn't betray the extra dimension's existence. Big beings in the garden-hose universe would fill out the entire second, small dimension and would never know that this dimension was there. Without the ability to detect structure or variations along the extra dimension, such as wiggles or undulations of matter or energy, they could never register its existence. Any variations along the second dimension would be completely washed out, much as any variation in the thickness of a piece of paper on the scale of its atomic structure is something you don't ever notice.

The two-dimensional world in which the dreaming Athena found herself was very much like the garden-hose universe. Because Athena had the opportunities to be both big and small relative to TwoDLand's width, she could observe this universe from both the perspective of someone bigger and that of someone smaller than its second dimension. To the big Athena, TwoDLand and OneDLand appeared the same in every respect. Only the small Athena could tell the difference. Similarly, in the garden-hose universe a being would be ignorant of an additional spatial dimension if it were too tiny for it to see.

Let's now return to the Kaluza-Klein universe, which has the three spatial dimensions we know about, supplemented by an extra one that's unseen. We can again use Figure 16 to think about this situation. Ideally, I would draw four spatial dimensions, but unfortunately that's

not possible (even a pop-up book wouldn't suffice). However, since the three infinite dimensions that constitute our space are all qualitatively the same, I really need only draw just one representative dimension. That leaves me free to use the other dimension to represent the unseen extra dimension. The other dimension shown here is the one that's curled up—the one that's fundamentally different from the other three.

Just as with our two-dimensional garden-hose universe, a four-dimensional Kaluza-Klein universe with a single tiny, rolled-up dimension would appear to us to have one dimension fewer than the four it actually has. Because we wouldn't know about the additional spatial dimension unless we could detect evidence of structure on its minute scale, the Kaluza-Klein universe would look three-dimensional. Rolled-up, or compactified, extra dimensions will never be detected if they are sufficiently tiny. Later on, we'll investigate just *how* tiny, but for now, rest assured that the Planck length is well below the threshold of detectability.

In life, and in physics, we only register those details that actually matter to us. If you cannot observe detailed structure, you might as well pretend it isn't there. In physics, this disregard of local detail is embodied in the effective theory idea of the previous chapter. In an effective theory, all that matters are the things that you can actually perceive. In the example above, we would use a three-dimensional effective theory where the information about extra dimensions is suppressed.

Although the curled-up dimension of the Kaluza-Klein universe is not far away, it's so small that any variation within it is imperceptible. Just as differences among New Yorkers don't really matter to people outside, the structure in the extra dimensions of the universe is irrelevant when its details vary on such minuscule a scale. Even if fundamentally there turn out to be many more dimensions than we acknowledge in our daily lives, everything we see can still be described in terms of only the dimensions we observe. Extremely small extra dimensions change nothing about the way we view the world, or even about how we do most physics calculations. Even if additional dimensions exist, if we are incapable of seeing or experiencing them, we can ignore them and still correctly describe what we see. Later on

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we'll see modifications to this simple picture for which this won't always be true, but those will involve additional assumptions.

We can understand one further important point about a rolled-up dimension from Figure 17, which illustrates the hose, or universe with one dimension, rolled up into a circle. Focus on any point along the infinite dimension. Notice that at each and every point there sits the entire compact space, namely the circle. The hose consists of all these circles glued together, like the slices I talked about in Chapter 1.

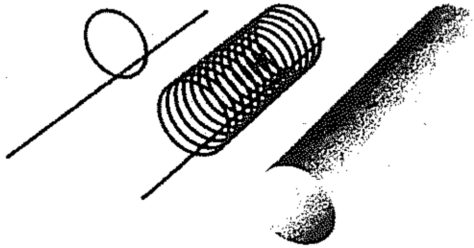

Figure 17. *In a two-dimensional universe, when a dimension is curled up there is a circle at every point along the infinite dimension of space.*

Figure 18 presents a different example: here there are two infinite dimensions rather than one, plus a single additional dimension curled up into a circle. In this case, there is a circle at each and every

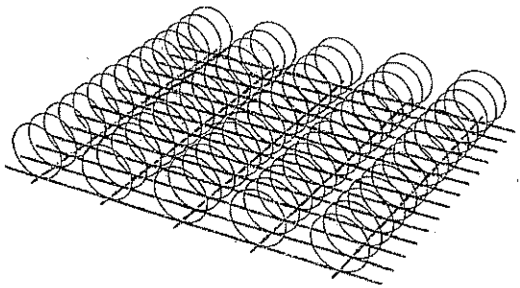

Figure 18. *In a three-dimensional universe, if one of the three dimensions is curled up you have a circle at every point in the plane.*

point in the two-dimensional space. And if there were three infinite dimensions, the rolled-up dimensions would exist at every point in three-dimensional space. You might liken the points in extra-dimensional space to the cells in your body, each of which carries your entire DNA sequence. Similarly, each point in our three-dimensional space could host an entire compactified circle.

So far, we've only considered a single additional dimension, which is rolled up into a circle. But everything we've said would hold true even if that curled-up dimension took some other shape—any shape at all. And it would also be true if there were two or more tiny, rolled-up dimensions of any shape at all. Any and all dimensions that are sufficiently small would be completely invisible to us.

Let us consider an example with two rolled-up dimensions. There are many possible shapes that these rolled-up dimensions could take. We'll choose a *torus*, a donut-like shape in which the two additional dimensions are both simultaneously rolled into a circle. This is illustrated in Figure 19. If both circles—the one that winds through the donut hole and the one that winds around the donut itself—are sufficiently small, the additional two rolled-up dimensions would never be seen.

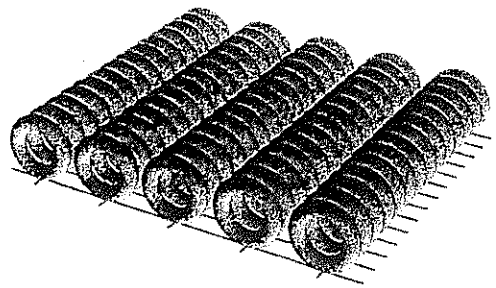

Figure 19. *When two out of four dimensions are curled into a donut, you have a donut at every point in space.*

But that's just one example. With more dimensions there are a huge number of conceivable *compact spaces*—spaces with rolled-up dimensions, distinguished by the precise manner in which the dimensions are rolled up. One category of compact spaces important to string theory are the *Calabi-Yau manifolds*, named after the Italian mathematician Eugenio Calabi, who first proposed these particular shapes, and the Chinese-born Harvard mathematician Shing-Tung Yau, who showed that they are mathematically possible. These geometric shapes roll up and wind together extra dimensions in a very special way. The dimensions are curled up into a small size, as with all compactifications, but they are tangled in a way that is more complicated and difficult to draw.[4]

Whatever shape the rolled-up extra dimensions take, and however many there are, at each point along the infinite dimensions there would be a small compact space containing all the curled-up dimensions. So, for example, if string theorists are right, everywhere in visible space—at the tip of your nose, at the North Pole of Venus, at the spot above the tennis court where your racket hit the ball the last time you served—there would be a six-dimensional Calabi-Yau manifold of invisibly tiny size. The higher-dimensional geometry would be present at every point in space.

String theorists often suggest—as Klein did—that curled-up dimensions are as small as the Planck length, 10^{-33} cm. Planck-length-size compact dimensions would be extraordinarily well hidden; there is almost certainly no way for us to detect something so small. Therefore, Planck-length extra dimensions would very likely leave no visible trace of their existence. So even if we live in a universe with Planck-length extra dimensions, we would still register only the three familiar dimensions. The universe could have many such tiny dimensions, but we might never have the resolving power to find out.

Newton's Gravitational Force Law with Extra Dimensions

It is nice to have a pictorial, descriptive explanation for why the extra dimensions are hidden when compactified, or rolled up, to a very minute size. But it's a good idea to check that the laws of physics accord with this intuition.

Let's take a look at Newton's gravitational force law, the well-

established form of the gravitational force law that Newton proposed in the seventeenth century. This law tells us how the gravitational force depends on the distance between two massive objects.* It's known as an *inverse square law*, which means that the strength of gravity decreases with distance proportionally to the distance squared. For example, if you double the distance between two objects, the strength of their gravitational attraction goes down by a factor of four. If the separation is increased to three times its original value, gravitational attraction decreases by a factor of nine. The inverse square law of gravity is one of the oldest and most important laws of physics. Among other things, it is the reason that planets have the type of orbits they do. Any viable physical theory of gravity must reproduce the inverse square law or it would be bound to fail.

The way in which the gravitational force law depends on distance, which is encoded in Newton's inverse square law, is intimately connected to the number of spatial dimensions. This is because the number of dimensions determines how quickly gravity diffuses as it spreads out in space.

Let's reflect on the connection, which will be very relevant to us later on when we consider extra dimensions. We'll do this by imagining a water supply whose water can be directed through either a hose or a sprinkler. We'll assume that both the hose and the sprinkler have the same amount of water running through them, and that they can each water a certain flower in a garden (see Figure 20). When the water goes through the hose, which is directed at the flower, the flower will get all the water. The distance from the base of the hose to the nozzle directed at the flower is irrelevant, because all the water must end up on the flower, no matter how far away the hose happened to be.

However, suppose instead that the same water is directed through a sprinkler that simultaneously waters many flowers. That is, the sprinkler sends out water in a circle, reaching all the flowers a certain distance away. Because the water will now be distributed among everything at that distance, the original flower will no longer get all

*In this book a "massive" object means an object with mass. A massive object is to be distinguished from a "massless" object, which has zero mass (and travels at the speed of light).

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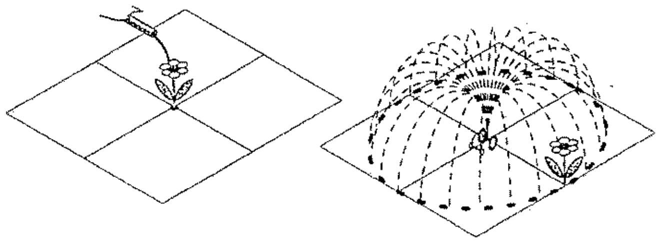

Figure 20. *The amount of water delivered to a flower by a sprinkler that sprinkles water around a circle is less than the amount delivered directly by a hose.*

the water. Moreover, the farther away the flower is from the source, the more greenery the sprinkler will water, and the more widely distributed the water will be (see Figure 21). That's because you can fit more plants on a circle three meters in circumference, say, than a circle just one meter in circumference. Because the water is more widely spread out, a farther away flower receives less water.

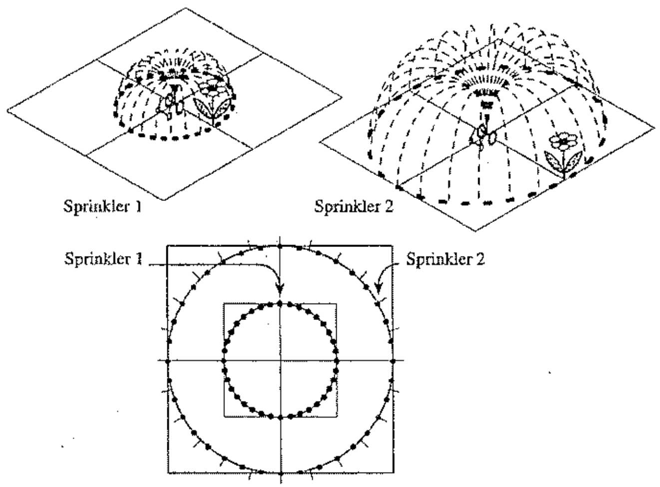

Figure 21. *When a sprinkler delivers water around a circle of larger radius, the water is spaced out more and the flower receives less water.*

Similarly, anything that is shared uniformly in more than one direction will have a smaller impact on any particular thing that is farther away—whether that thing is a flower or, as we will soon see, an object experiencing the force of gravity. Gravity, like water, is more widely distributed when it is farther away.

With this example, we can also see why the distribution depends so strongly on the number of dimensions in which water (or gravity) is spread. The water from the two-dimensional sprinkler is spread out with distance, unlike the water from the one-dimensional hose, which is not spread out at all. Now imagine a sprinkler that spreads its water over the surface of a sphere, and not just around a circle. (Such a sprinkler would look something like a dandelion gone to seed.) Here, the water will spread out with distance much more quickly.

Let's now apply this reasoning to gravity, and derive the precise distance dependence of the gravitational force in three dimensions. Newton's gravitational force law follows from two facts: that gravity acts equally in all directions, and that there are three dimensions of space. Let's now imagine a planet, which attracts any mass in its vicinity. Because the gravitational force is the same in all directions, the strength of the gravitational attraction that the planet exerts on another massive object—a moon, for example—will depend not on direction, but on the distance between them.

To pictorially represent the strength of the gravitational force, the left of Figure 22 shows radial lines extending outwards from the planet's center, resembling water spreading out from a sprinkler. The density of these lines determines the strength of gravitational attraction that the planet exerts on anything in its vicinity. More force

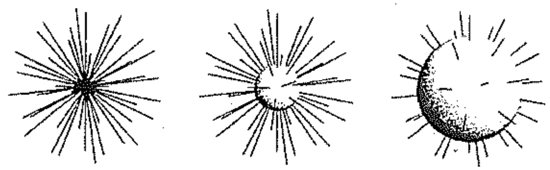

Figure 22. *Gravitational force lines emitted from a massive object, such as a planet. The same number of lines intersect a sphere of any radius; therefore, the force lines are more diffuse and gravity is weaker the farther you are from the massive object at the center.*

lines passing through an object would mean a greater gravitational attraction, and fewer force lines would mean a smaller gravitational attraction.

Notice that the same number of force lines intersect a spherical shell drawn any distance away, no matter how far or near (center and right of Figure 22). The number of force lines never changes. But because the force lines are spread-out among all the points on the sphere's surface, the force at a greater distance is necessarily weaker. The precise dilution factor is determined by the quantitative measure of how widely distributed the force lines are at any given distance.

A fixed number of force lines passes through a sphere's surface, whatever its distance from the mass. The area of that sphere's surface is proportional to its radius squared: the surface area is equal to a number multiplied by the square of the radius. Because the fixed number of gravitational force lines is spread out over the sphere's surface, the gravitational force has to decrease as the square of the radius. This spreading out of the gravitational field is the origin of the inverse square law for gravity.

Newton's Law with Compact Dimensions

So we now know that in three dimensions, gravity should obey an inverse square law. Notice that the argument seems to depend critically on the fact that there are three spatial dimensions. Had there been only two dimensions, gravity would have been spread out only over a circle, and the force of gravity would have decreased with distance at a slower rate. Had there been more than three dimensions, the surface area of a hypersphere would have grown far more rapidly with the separation between the planet and its moon, and the force would have fallen off that much more quickly. It seems that only three spatial dimensions yields the inverse square distance dependence. But if that is the case, how can theories with extra dimensions yield Newton's inverse square law for gravity?

It is very interesting to see how compactified dimensions resolve this potential conflict. The essence of the logic is that force lines cannot spread arbitrarily far into the compact dimensions because those com-

pact dimensions have finite size. Although force lines initially spread out in all dimensions, when they have spread beyond the extra dimensions' sizes they have no choice but to spread out solely in the directions of the infinite dimensions.

This can be illustrated once again with our hose example. Imagine that water enters the hose through a small pinhole in a cap covering the end of the hose (see Figure 23). Water directed through the puncture will not immediately travel directly down the hose, but will first spread throughout the tube's cross-section. Nonetheless, it should be clear that if you were at the other end of the hose, watering your flower, the way the water entered would make no difference at all. Although the water would first spread in more than one direction, it would quickly reach the inside surface of the hose and flow once again as if there were only one direction. This is essentially what happens to gravitational field lines in small, compactified dimensions.

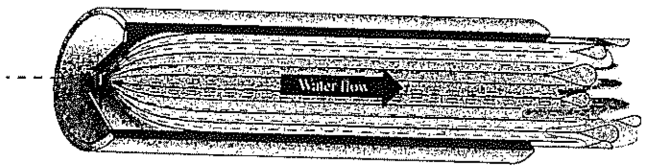

Figure 23. *Water entering a garden hose through a pinhole at the end first spreads in three dimensions before traveling only along the single long dimension of the hose.*

As before, we can imagine a fixed number of force lines emanating from a massive sphere. At a distance smaller than the extra dimensions' size, these force lines will spread out equally in all directions. If you could measure gravity on that small scale, you would measure the consequences of higher-dimensional gravity. The force lines would spread the way water does as it enters the hose through the pinhole and spreads throughout the hose's interior.

However, at distances greater than the extra dimensions' sizes, the force lines can spread only in the infinite directions (see Figure 24). In the small, compact dimensions, the force lines will hit the edge of space, so they can't spread out any farther that way. They have to bend, and the only way for them to go is in the direction of the large

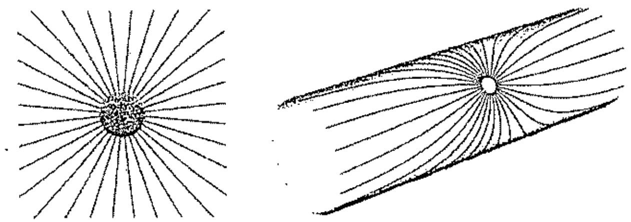

Figure 24. *Gravitational force lines emitted from a massive object when a dimension is curled up. The force lines spread radially over short distances, but over long distances they extend only along the infinite dimension.*

dimensions. Therefore, at distances greater than the sizes of the extra dimensions, it's just as if the extra dimensions didn't exist, and the force law reverts to Newton's inverse square law—the one we observe. This means that even from a quantitative point of view, you won't know there are extra dimensions if you measure the gravitational force only between objects with separations greater than the curled-up dimensions' size. The distance dependence reflects extra dimensions only in the tiny region inside the compact space.

Other Ways to Bound Dimensions?

We've now established that when extra dimensions are sufficiently small, they are invisible and have no detectable consequences on the length scales we observe. For a long time, string theorists assumed that extra dimensions were Planck-length dimensions, but recently some of us have questioned this assumption.

No one understands string theory well enough to say definitively what the sizes of extra dimensions will turn out to be. Sizes comparable to the Planck length are possible, but any dimension too small to observe is also in the running. The Planck length is so tiny that even considerably larger curled-up dimensions might well escape notice. An important question for the study of extra dimensions is just how big these dimensions can be, given that we haven't seen them yet.

The questions we'll address in this book include how big extra

dimensions can be, whether these dimensions have any discernible effect on elementary particles, and how experiments might probe them. We will see that the existence of extra dimensions can significantly change the rules by which we do particle physics and, furthermore, that some of these changes will have experimentally observable consequences.

An even more radical question we'll investigate is whether additional dimensions have to be small. We don't see tiny dimensions, but do dimensions have to be small to be invisible? Could an extra dimension possibly extend for ever without our seeing it? If so, extra dimensions would have to be very different from the dimensions we've looked at. So far I've presented only the simplest possibility. We'll see later why even the radical possibility of an infinite extra dimension cannot be excluded if it is sufficiently different from the three familiar infinite dimensions.

The next chapter will address yet another question that might have occurred to you: why can't small extra dimensions just be intervals, not curled up into a ball but instead bounded between two "walls"? This possibility didn't occur to anyone right away—but why not? The reason is that imagining an end to space entails knowing what is happening there. Would things fall off the end of the universe, as old pictures of the flat Earth seemed to imply? Or would they be reflected back? Or would they never get there? The need to specify what would happen at the end means that you have to know what scientists call *boundary conditions*. If space ends, where and on what does it end?

Branes—membrane-like objects in higher-dimensional space—provide the necessary boundary conditions for worlds that "end." As we will see in the following chapter, branes can make a world (or many worlds) of difference.

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Exclusive Passages: Branes, Braneworlds, and the Bulk

I'm gonna stick like glue,
Stick, because I'm stuck on you.

Elvis Presley

Unlike the studious Athena, Ike rarely read any books. He generally preferred playing with games, gadgets, and cars. But Ike hated driving in Boston, where the drivers were reckless, the roads were badly signposted, and the highways were invariably under construction. Ike always ended up stuck in traffic, which he found especially frustrating when he could see a nearly empty freeway overhead. Though the empty road would be tempting, Ike would have no way to quickly reach it since, unlike Athena's owls, he couldn't fly. For Ike trapped on slow roads in Boston, the third dimension was no use at all.

Until very recently, few self-respecting physicists considered extra dimensions worth thinking about. They were too speculative and too foreign: no one could say anything definitive about them. But in the last few years, extra dimensions have found their fortunes rising. No longer shunned as undesirable gatecrashers, they've evolved into highly sought-after, stimulating company. They owe their newfound respectability to branes and to the many genuinely new theoretical possibilities that these fascinating constructs have introduced.

Branes took the physics community by storm in 1995, when the physicist Joe Polchinski of the Kavli Institute for Theoretical Physics

(KITP) in Santa Barbara established that they were essential to string theory. But even before then, physicists had proposed branelike objects. One such example was a *p-brane* (so called by p-layful p-hysicists), an object that extends infinitely far in only some dimensions, which physicists derived mathematically using Einstein's theory of general relativity. Particle physics had also suggested mechanisms for confining particles on branelike surfaces. But string theory branes were the first known type of brane that could trap forces as well as particles, and we'll soon see that is part of what makes them so interesting. Like Ike stuck on a two-dimensional road in three-dimensional space, particles and forces can be trapped on lower-dimensional surfaces called branes, even if the universe has many other dimensions to explore. If string theory accurately describes the world in which we live, physicists have no choice but to acknowledge the potential existence of such branes.

The world of branes is an exciting new landscape that has revolutionized our understanding of gravity, particle physics, and cosmology. Branes might really exist in the cosmos, and there is no good reason that we couldn't be living on one. Branes might even play an important role in determining the physical properties of our universe and ultimately explain observable phenomena. If they do, branes and extra dimensions will be here to stay.

Branes as Slices

In Chapter 1 we looked at one way of thinking about the two-dimensional world of Flatland: as a two-dimensional slice of a three-dimensional space. In Abbott's novel, the character A. Square took a journey beyond two-dimensional Flatland, into the third dimension, and recognized that Flatland was a mere slice of the bigger three-dimensional world.

Upon his return, A. Square suggested—logically enough—that the three-dimensional world he had seen might also be a mere slice: a three-dimensional slice of an even higher-dimensional space. By "slice," of course, I don't mean merely a paper-thin, two-dimensional membrane, but the logical extension of such a thing—a generalized

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membrane, if you like. You might think of the three-dimensional slices that A. Square suggested as three-dimensional chunks in four-dimensional space.

But his three-dimensional guide promptly dismissed A. Square's speculation about three-dimensional slices. Like almost everyone we know, this unimaginative inhabitant of three dimensions believed in only the three dimensions of space he could see; he couldn't even contemplate a fourth.

Branes have introduced mathematical notions into physics that are similar to those described in *Flatland* over a century ago. Physicists have now returned to the idea that the three-dimensional world that surrounds us could be a three-dimensional slice of a higher-dimensional world. A brane is a distinct region of spacetime that extends through only a (possibly multidimensional) slice of space. The word "membrane" motivated the choice of the word "brane" because membranes, like branes, are layers that either surround or run through a substance. Some branes are "slices" inside the space, but others are "slices" that bound space, like slices of bread in a sandwich.

Either way, a brane is a domain that has fewer dimensions than the full higher-dimensional space that surrounds or borders it.[5] Note that membranes have two dimensions, but branes can have any number of dimensions. Although the branes that will most interest us have three spatial dimensions, the word "brane" refers to all "slices" of this sort; some branes have three spatial dimensions, but other branes have more (or fewer).[6] We'll use *3-branes* to refer to branes with three dimensions, *4-branes* to refer to those with four, and so on.

Boundary Branes and Embedded Branes

In the previous chapter I explained why we might not see extra dimensions. They could be curled up into sizes so small that evidence of their existence never would appear. The key point was that the extra dimensions would be small. None of the reasons for the invisibility of dimensions relied on the fact that extra dimensions were curled up.

This suggests an alternative possibility: perhaps dimensions are not rolled up, but simply terminate within a finite distance. Because

dimensions that disappear into nothing are potentially dangerous—you wouldn't want pieces of the universe to fall off the ends—there must be boundaries for the finite dimensions that tell them where and how to end. The question is, what happens to particles and energy when they reach these boundaries?

The answer is that they encounter a brane. In a higher-dimensional world, branes would be the boundaries of the full higher-dimensional space, known as the *bulk*. Unlike a brane, the bulk extends in all directions. The bulk spans every dimension, both on and off the brane (see Figure 25). The bulk is therefore "bulky," whereas, in comparison, the brane is flat (in some dimensions), like a pancake. If branes bordered the bulk in certain directions, some of the bulk's dimensions would be parallel to the brane, while other dimensions would lead off it. If the brane is the boundary, the dimensions off the brane would extend only to one side.

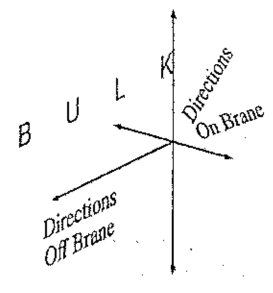

Figure 25. *A brane is a lower-dimensional surface with directions along it and directions that lead away from it, into the higher-dimensional bulk.*

To understand the nature of finite dimensions that end on branes, let us consider a very long thin pipe. Within the pipe there are three dimensions: one long and two short. To make the analogy to flat branes most straightforward, let's imagine that our pipe has a square cross-section. An infinitely long pipe of this type would have four infinitely long straight walls. If the pipe were a universe in its own

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right, it would be one with three dimensions, two of which are bounded on either side by walls and one that extends infinitely far.

We know that a long thin pipe when viewed from afar (or with insufficient resolution) looks one-dimensional, much like the garden hose of the previous chapter. But we can also ask, as we did before with the garden-hose universe, how the pipe universe—consisting of the pipe and its interior—would appear to a conscious being living inside.

As you might suspect, this would depend on the being's resolution. A small fly that could move around within the square pipe would experience it as three-dimensional. Unlike the two-dimensional garden-hose example, we are assuming that the fly can move inside the pipe, and not just on its exterior. Nonetheless, as with the garden hose, the fly would experience the one long dimension differently than the other two. In one direction the fly could go arbitrarily far (assuming that our pipe is very long or infinite), whereas in the other two directions the fly could only go a short distance—the width of the pipe.

But there is a difference between the garden-hose universe and the pipe universe, aside from the number of dimensions each has. Unlike the bug of the previous chapter, the fly in the pipe travels inside it. Thus the fly sometimes encounters walls. It can go back and forth, or up and down, and reach a boundary. The bug on the hose, on the other hand, would never reach such a boundary: instead, it would only go round and round.

When the fly reaches the boundary of its pipe universe, there have to be rules that govern how it behaves. The walls of the pipe determine that behavior. The fly might hit the wall and splat into it; or the pipe might be reflective, so that the fly bounces off. If the pipe were a true universe bounded by branes, then the branes, which would be two-dimensional, would determine what happens when a particle, or anything else that could carry energy, reaches them.

When things get to a boundary brane, they bounce back, just as billiard balls bounce from the edges of the table or light bounces back from a mirror. This is an example of what physicists call a *reflective boundary condition*. If things bounce back from a brane, energy is not lost; it doesn't get absorbed in the branes or leak away. Nothing goes beyond the branes. The boundary branes are the "ends of the world."

In a multidimensional universe, branes serve the role of the boundary walls in the pipe-universe example above. Like walls, such branes would have lower dimension than the full space; a boundary always has a lower dimension than the object it bounds. That is as true for the boundary of space as it is for the crust that is the boundary of a loaf of bread. It is also true for the walls in your house, which have one lower dimension than the room they enclose: the room is three-dimensional, whereas any individual wall (when we ignore its thickness) spans only two dimensions.

Although so far in this section I have concentrated on branes that sit at boundaries, branes don't always sit at the edge of the bulk. They could conceivably exist anywhere in space. In particular, branes might sit somewhere away from the boundary, inside of space. If a boundary brane is like a thin heel at the end of a loaf of bread, such a non-boundary brane would be like a thin slice of bread within the loaf. A non-boundary brane would still be a lower-dimensional object, like the ones we have already considered. But non-boundary branes would have higher-dimensional bulk space on either side.

In the next section, we'll see that whatever the number of dimensions of the bulk or of the brane, and no matter whether branes are inside a space or at a boundary, branes can trap particles and forces along them. This makes the region of space they occupy very special.

Trapped on Branes

It is very unlikely that you will explore all the space available to you. There are probably places that you wish you had visited and voyages you'll never take—into outer space or the depths of the sea, for example. You haven't been to these places, but, in principle, you could go. There is no physical law that makes it impossible.

If, however, you lived inside a black hole, your travel opportunities would be far more severely constrained, more restricted even than those of women in Saudi Arabia. The black hole (until it decayed away) would keep you (or rather, the mutilated, black hole version of you) trapped in the interior, and you would never be able to escape.

There are many more familiar examples of things with restricted freedom of movement for which there are regions of space that are truly inaccessible. A charge on a wire and a bead on an abacus are both objects that live in a three-dimensional world, but travel in only one of its dimensions. There are also commonplace things that are confined to two-dimensional surfaces. Water droplets on a shower curtain travel only along the curtain's two-dimensional surface (see Figure 26). Bacteria trapped between microscope slides also experience only two-dimensional motion. Another example is Sam Loyd's "fifteen" game, the annoying game consisting of a little plastic tray with letters on tiles that you push around until they are correctly arranged in a square and say something like LOOK/YOUF/INIS/HED (see Figure 27). Unless you cheat, the letters stay within their plastic enclosure; they can never move in a third dimension.

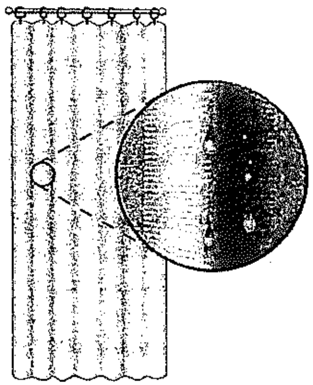

Figure 26. *Drops of water stuck on a two-dimensional shower curtain in a three-dimensional room.*

Branes, like shower curtains and Loyd's fifteen game, trap things on lower-dimensional surfaces. They introduce the possibility that in a world with additional dimensions, not all matter is free to travel everywhere. Just as the water droplets on the curtain are bound to a two-dimensional surface, particles or strings can be confined to a three-dimensional brane sitting inside a higher-dimensional world. But unlike the droplets on the curtain, they are truly trapped. And

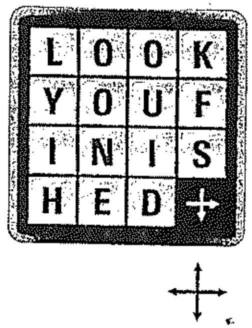

Figure 27. *Sam Loyd's "fifteen" game.*

unlike the fifteen game, branes are not arbitrary. They are natural players in a higher-dimensional world.

Particles confined to branes are truly trapped on those branes by physical laws. Brane-bound objects never venture into the extra dimensions that extend off the brane. Not all particles will be trapped on branes; some particles might be free to travel throughout the bulk. But what distinguishes theories with branes from multidimensional theories without them are the particles on the branes—the ones that don't travel through all the dimensions.

In principle, branes and the bulk could have any number of dimensions, so long as a brane never has more dimensions than the bulk. The *dimensionality of a brane* is the number of dimensions in which brane-confined particles are permitted to travel. Although there are many possibilities, the branes that will be most interesting to us later on will be the three-dimensional ones. We don't know why three dimensions should appear to be so special. But branes with three spatial dimensions could be relevant to our world because they could extend along the three spatial dimensions we know. Such branes could appear in a bulk space with any number of dimensions that is more than three—four, five, or more dimensions.

Even if the universe does have many dimensions, if the particles and forces with which we are familiar are trapped on a brane that extends in three dimensions, they would still behave as if they lived in only three. Particles confined to branes would travel only along the brane.

And if light were also stuck to the brane, light rays would spread out only along the brane. In a three-dimensional brane, light would behave exactly as it would in a truly three-dimensional universe.

Furthermore, forces trapped on a brane influence only particles confined to this same brane. The material of which we are composed, such as nuclei and electrons, and the forces through which these building blocks interact, such as the electric force, might be confined on a three-dimensional brane. Brane-bound forces would spread out only along their brane, and brane-bound particles would be exchanged and would travel solely along the dimensions of the brane.

So if you lived in such a three-dimensional brane, you would be able to travel freely along its dimensions, much as you do in three dimensions now. Anything confined within a three-dimensional brane would look just the same as it would if the world were truly three-dimensional. The other dimensions would exist adjacent to the brane, but things stuck to a three-dimensional brane would never penetrate the higher-dimensional bulk.

But although forces and matter can be stuck on a brane, braneworlds are interesting precisely because we know that not everything is confined to a single brane. Gravity, for example, is never confined to a brane. According to general relativity, gravity is woven into the framework of space and time. That means that gravity must be exerted throughout space and in every dimension. If it could be confined to a single brane, we would have to abandon general relativity.

Fortunately this is not the case. Even if branes exist, gravity will be felt everywhere, on and off branes. This is important because it means that braneworlds have to interact with the bulk, even if only via gravity. Because gravity extends into the bulk, and everything interacts via gravity, braneworlds will always be connected to the extra dimensions. Braneworlds do not exist in isolation: they are part of a larger whole with which they interact. In addition to gravity, there could conceivably exist other particles and forces in the bulk. If there are, such particles could also interact with particles confined to a brane and connect brane-bound particles to the higher-dimensional bulk.

The string theory branes that we will briefly consider later on have specific properties aside from the ones I have mentioned: they can carry particular charges, and they will respond in particular ways

when something pushes on them. However, I will rarely bring in such detailed properties later on when I talk about branes. It will be enough to know the properties we have considered in this chapter: branes are lower-dimensional surfaces that can house forces and particles, and they can be the boundaries of higher-dimensional space.

Braneworlds: Blueprints for a Jungle Gym of Branes

Because branes could trap most particles and forces, the universe we live in could conceivably be housed on a three-dimensional brane, floating in an extra-dimensional sea. Gravity would extend into the extra dimensions, but stars, planets, people, and everything else that we sense could be confined to a three-dimensional brane. We would then be living on a brane. A brane might be our habitat. The concept of braneworlds is based on this assumption (see Figure 28).

If there can be one brane suspended in a higher-dimensional spacetime, there is no denying the possibility of many more. Braneworld scenarios often involve more than a single brane. We don't yet

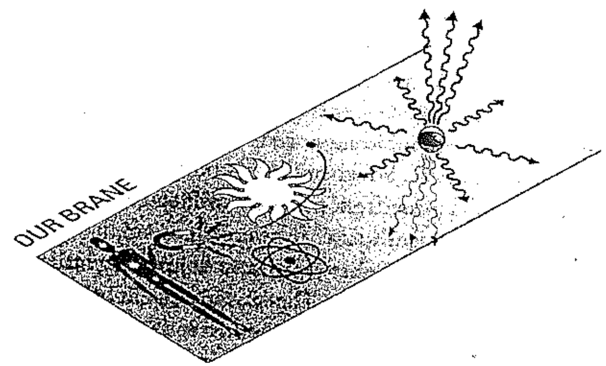

Figure 28. *We could be living on a brane. That is, the matter we are made of, photons, and other Standard Model particles can all be on the brane. But gravity is always everywhere—on the brane and in the bulk, as is illustrated by the squiggly lines.*

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know the number or types of branes that could be present in the cosmos. *Multiverse* is a name that is sometimes attached to theories with more than one brane (see Figure 29). People often use the word to describe a cosmos with non-interacting or only weakly interacting pieces.

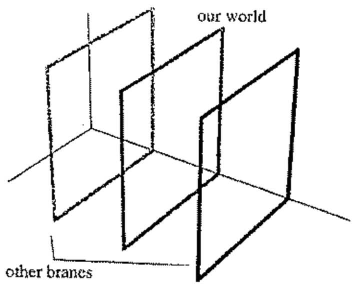

Figure 29. *The universe can contain multiple branes that interact only via gravity or don't interact at all. Such set-ups are sometimes called multiverses.*

I find the term "multiverse" a bit strange, since a universe is defined as the whole that is the unity of its parts. It is possible, however, to have different branes that are too far apart ever to communicate with one another, or that can communicate with one another only weakly, through mediating particles that travel between them. Particles on distant branes, then, would experience entirely different forces, and brane-bound particles would never have direct contact with particles bound on another brane. So when there is more than one brane with no force in common aside from gravity, I will sometimes refer to the universe housing them both as a multiverse.

Thinking about branes makes you aware of just how little we know about the space in which we live. The universe might be a magnificent composition linking intermittent branes. Even if we know the basic ingredients, in a multiverse populated by more than one brane, exotic new scenarios for the geometry of space are conceivable as well as myriad possibilities for how the particles we know and don't know are distributed among them. A single deck of cards can yield many different hands. There are scores of possibilities.

Other branes might be parallel to ours and might house parallel worlds. But many other types of braneworld might exist too. Branes could intersect and particles could be trapped at the intersections. Branes could have different dimensionality. They could curve. They could move. They could wrap around unseen invisible dimensions. Let your imagination run wild and draw any picture you like. It is not impossible that such a geometry exists in the cosmos.

In a world in which branes are embedded in a higher-dimensional bulk, there could be some particles that explore the higher dimensions and others that stay trapped on branes. If the bulk separates one brane from another, some particles can be on the first brane, some on the other, and some in the middle. Theories tell us about many ways in which particles and forces might be distributed among different branes and the bulk. Even for branes derived from string theory, we don't yet know why string theory should single out any particular allocation of particles and forces. Braneworlds introduce new physical scenarios that might describe both the world we think we know and other worlds we don't know on other branes we don't know, separated from our world in unseen dimensions.

New forces confined to distant branes might exist. New particles with which we will never directly interact might propagate on such other branes. Additional stuff accounting for dark matter and dark energy—the matter and energy that we surmise from their gravitational effects but whose identity is a mystery—might be distributed among different branes, or even in both the bulk and on other branes. And gravity might even influence particles differently as you go from one brane to the next.

If there is life on another brane, those beings, imprisoned in an entirely different environment, most likely experience entirely different forces that are detected by different senses. Our senses are attuned to the chemistry, light, and sound surrounding us. Because fundamental forces and particles are likely to be different, the creatures of other branes, should they exist, are unlikely to bear much resemblance to the life of our brane. The other branes will probably be nothing like our own. The only necessarily shared force is gravity, and even gravity's influence can vary.

The consequences of a braneworld will depend on the number and

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types of branes, and where they are located. Unfortunately for the curious, particles and forces confined to distant branes are not required to influence us very strongly. They might merely determine what travels in the bulk, and emit weak signals which might never even reach us. Therefore many conceivable braneworlds will be very difficult to detect, even if they do exist. After all, gravity is the only interaction that we know for sure is shared between the stuff on our brane and the stuff on any other brane, and gravity is an extremely weak force. Without direct evidence, other branes will remain cloistered in the realm of theory and conjecture.

But some of the braneworlds I will present could lead to detectable signals. The detectable braneworlds are the ones that have implications for the physical features of our world. Even though the proliferation of possible braneworlds is in some respects frustrating, it is really quite exciting. Not only might branes help resolve long-standing problems in particle physics, but if we're lucky, and one of the scenarios that I will describe is correct, evidence for braneworlds should appear in experiments with elementary particle physics very soon. We might really be living on a brane—and we might actually know it within a decade.

As of now, we do not know which, if any, of the many possibilities is the true description of the universe. I will therefore keep all options open, so as not to omit anything interesting. Whatever scenario turns out to describe our world, the ones I will present introduce new and fascinating ideas that no one would previously have thought possible.

4

Approaches to Theoretical Physics

She's a model and she's looking good.

Kraftwerk

"Hey, Athena, is that Casablanca *you're watching?"*

"Sure is. Want to join me? This is such a great scene."

You must remember this,
A kiss is just a kiss.
A sigh is just a sigh.
The fundamental things apply as time goes by.

"Hang on, Ike. Don't you think that last line's a little weird? It's supposed to be so romantic, but it almost sounds as if it's about physics."

"Athena, if you think that's strange, you've got to hear the opening verse of the original:"

This day and age we're living in
Give cause for apprehension,
With speed and new invention.
And things like fourth dimension,
Yet we get a trifle weary
With Mr. Einstein's theory . . .

"Ike, you don't really expect me to believe that, do you? Next thing I know you'll tell me Rick and Ilsa escape into the seventh dimension! Why don't we forget I ever said anything and just sit back and watch the movie?"